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A Robust Flexible Stochastic Mathematical Model for Logistics Coordination in a Two-echelon Supply Chain

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ABSTRACT

Uncertainty approaches have been studied by many scholars. What makes them more interesting is the combination of a few approaches in order to make stronger and more flexible mathematical models. In this paper, we have proposed a robust flexible stochastic mathematical model which combines the robust optimization techniques, fuzzy soft constraint approach, and scenario-based stochastic approach. The created approach can provide a framework for better and simultaneous tackling different types of uncertainties in mathematical models. We have embedded the given general model into a bi-level supply chain problem, called location-inventory-routing problem. In this problem, we have considered a two-echelon supply chain consisting of some warehouses in the first echelon as leader and several retailers in the second echelon as follower. The supply chain tries to distribute a perishable item from the central warehouses to the retailers. The major uncertain parameters are demand of the retailers as well as some supply chain operational costs. The developed approach considers two different attitudes to uncertainty by the players in the game. We have employed the general fuzzy measures to implement the pessimistic-optimistic attitude of decision-makers. Furthermore, scenario-based robust programming techniques are utilized. A combination of fuzzy goal programming and multi-choice goal programming techniques are customized in order to solve the bi-level problem. A number of numerical samples with different sizes are solved and the results are analyzed. The results show that by increasing the pessimistic-optimistic attitude parameter of the leader, the leader's costs increases while by increasing the pessimistic-optimistic attitude parameter of the follower, the follower's total cost partially decreases.

Keywords: *supply chain coordination, location-inventory-routing, robust optimization, flexible programming, scenario-based programming*

1. INTRODUCTION

Today, uncertainty has become an integral part of supply chain (SC) issues for different reasons such as the spread of contagious diseases, wars, sanctions and global inflation. Uncertainty has a great impact on demand and costs, which play important roles in formulating SC problems. Facilities location, inventory control and vehicle routing are of the major problems and challenges of SCs, separately. Integration of these problems, specifically with the existence of uncertainty, makes it more challenging and complex. On the other hand, the existence of uncertainty in location-inventory-routing problem (LIRP) increases its complexity compared to each addressed single problem. In this research, we have tried to tackle the uncertainties in the LIRP.

Different tools have been used to tackle uncertainties in SC problems. Possibilistic or fuzzy-based approaches are mostly used when there are little information and the decision-making environment is ambiguous. In the mentioned approach, uncertainty can be handled by using fuzzy numbers and possibilistic, necessity and credibility measures. Stochastic approach is used when there is a history of required data and probabilities can be assigned for the

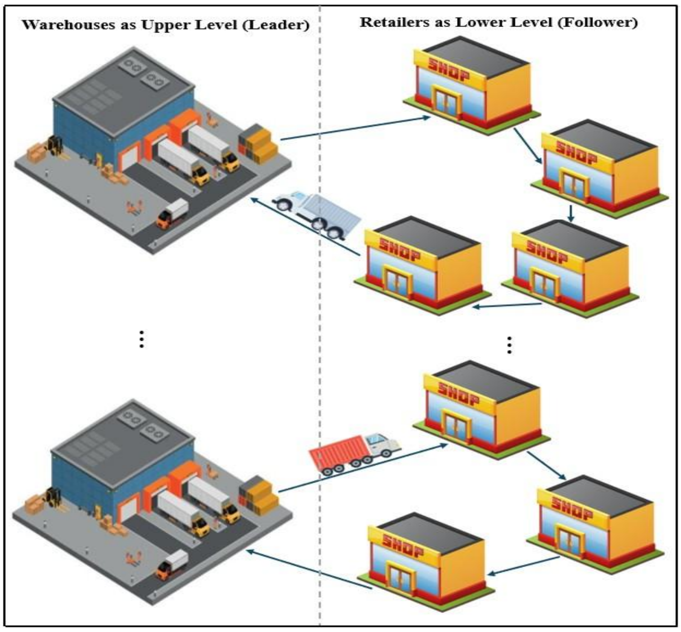

Figure 1 The Structure of the Investigated Decentralized SC with Two Echelons

occurrence of future events as in scenario-based programming. It should be noted that for some real-world situations, a combination of uncertainty tools should be applied in order to deal with various uncertainties. For example, some parameters like external demand should be expressed in the form of three optimistic, pessimistic, and possibilistic scenarios with specific probabilities and its value can be represented by using a fuzzy number for each scenario. This type of uncertainty is called hybrid uncertainty. In this research, we are going to tackle hybrid uncertainties in SC modeling.

In addition, most of the previous researches are concentrated on the application of uncertainty approaches in traditional centralized problems, while most of the real-world SC problems have more than one player or decision-maker (DM) who search for their own benefits. This is the case in the considered SC problem in this study; in fact, the LIRP model in this research is in the form of decentralized with two different levels, where a bi-level programming (BLP) approach is utilized. In the developed BLP model, there are two levels including upper level (UL) as the leader and lower level (LL) as the follower who make decisions for their own benefits. However, each level makes decisions separately, but each one considers the effects of other level's decisions on long-term collaborations. It should be noted that the UL and LL may have different pessimistic or optimistic attitudes in predicting the occurrence of uncertain events. Consideration of the two possible attitudes to uncertainty by the UL and the LL is one of the interesting contributions of the current research.

The under-study decentralized SC structure in this research includes a number of central warehouses in the first echelon and a given number of retailers in the second echelon. Warehouses are presumed to be handled by a central office called UL (i.e., leader of them game) while retailers are handled by another central office called the LL (i.e., follower). The aim of each level is to minimize its own operational costs. The costs involved with the UL include setting up new warehouses and transportation costs of the goods from warehouses to retailers by using some selected routes while cost involved with the LL is inventory holding cost. The decision variables of the UL are the locations of new warehouses and the routes to be selected in each period, while those of the LL are the amount of inventory held at each retailer and the amount of received shipments. Figure 1 shows the design of the described decentralized SC with two echelons or DMs.

An important assumption in this model is the perishability of distributed products to the retailers. Perishability is defined according to a defined storage life (or shelf life); in fact, products may be spoiled after a time greater than the shelf life. For example, if the shelf life of a product is three weeks, it means that the product may be spoiled after the three weeks. The proposed uncertain bi-level location-inventory-routing problem (BLLIRP) model can be employed to design the distribution network in many industries with perishable items such as food and pharmaceutical industries.

There are various methods to deal with uncertainty in optimization problems, such as stochastic, fuzzy, and robust programming. Robust programming/optimization is one of the most common and interesting methods in this area. A robust decision is a decision that can withstand the uncertainty of the problem so that the resulting solution has the least fluctuation. A solution to an optimization problem is a robust solution if it has both feasibility robustness and optimality robustness. Feasibility robustness means that the solution remains feasible for all (or almost all) possible values of uncertain parameter while optimality robustness means that the value of the objective function for a robust solution remains close to its optimal value for all (or almost all) possible values of the uncertain parameter. In this research, we have utilized the robust programming approach proposed by Pishvaee and Fazli Khalaf (2016) and Dehghan *et al.* (2018).

Generally, a new robust flexible stochastic model for the BLLIRP model with perishable product is proposed. The review of the literature shows that no study has proposed such flexible uncertain model for a BLP considering different attitudes of the DMs. Demand and costs are assumed to be uncertain. The used approaches or techniques for solving the problem includes fuzzy goal programming (FGP) method of Chen and Chen (2015) improved by multi-choice goal programming (MCGP) of Chang (2008); it is called FGP-MCGP which was developed by Partovi *et al.* (2023). By utilizing FGP-MCGP method, both the leader and follower can find their satisfactory solutions in just one stage instead of two time-consuming stages.

The research goals can be given as follows:

- Making Making the model of BLLIRP closer to the real-life situations for the leader and follower DMs considering different uncertainties.
- Making the model of BLLIRP closer to the real-life situations for the leader and follower DMs considering different uncertainties.
- Solving the addressed problem with different types of uncertainties in the model.
- Providing a situation in which different pessimistic and optimistic attitudes of the DMs are considered.

The research questions are:

1) How to consider different possible scenarios for the parameters of the BLLIRP model?
2) How to model and tackle flexible or soft constraints of the BLLIRP model?
3) How to provide a robust model for the DMs in the uncertain environment of the game?
4) How to solve the BLLIRP considering its bi-level nature?

The major novelties of this research are:

- Making the BLLIRP model closer to the real-life situations by considering different types of uncertainties.
- Proposing a combination of flexible, robust, and stochastic approaches for tackling uncertainties in the BLLIRP.
- Consideration of two different attitudes to uncertainty by the leader and follower.
- Solving the developed problem by customizing FGP-MCGP method.

The current article is structured as follows: we survey some pertinent researches in Section 2. We describe the notations of the proposed uncertain model in Section 3. We give descriptions about the solution method in Section 4. For showing the efficiency of the solution method, some numerical examples are designed and solved in Section 5. Eventually, we give the conclusions of this research and propose research ideas in Section 6

2. LITERATURE REVIEW

SC design has been of the focal attention of many researches, specifically in the uncertain environments such as Farahani *et al.* (2025). Rahmani Ahranjani *et al.* (2018) developed a stochastic robust approach for tackling uncertainty in a closed-loop SC design problem. Das *et al.* (2025) studied a three-echelon green SC in order to determine price and greening degree of a given product using possibility measure and game theory approach. Huang *et al.* (2025) studied distribution centers location in the international level using stochastic programming. Usefi *et al.* (2023) used robust possibilistic approach for the design of a relief SC. Furthermore, distribution center location problem not only is studied in commodities SCs but also in humanitarian SCc such as in Yılmaz and Kabak (2024). In this section, since the under-study problem is LIRP in the case of decentralization with uncertainty, we make a review on this problem with uncertainty in two centralized and decentralized situations.

2.1 Review of Centralized LIRP

A lot of researches have been done on LIRP under uncertainty. We review some of most pertinent ones in this section. Perhaps the first research conducted in this area is that of Ahmadi Javid and Azad (2010), in which the simultaneous optimization of facility location, inventory level and vehicle routing with uncertain demand was investigated and solved using a hybrid algorithm of simulated annealing (SA) and Tabu search (TS). Tavakkoli-Moghaddam and Raziei (2016) studied LIRP utilizing fuzzy logic in order to express the uncertainty of demand. Their proposed model was bi-objective, multi-period and multi-product. The First objective was to minimize construction, inventory and routing costs, and the second one was to minimize inventory shortages for customers. Zhalechian *et al.* (2016) proposed a new three-objective LIRP model for a closed-loop SC under mixed (or hybrid) uncertainty with environmental considerations. They considered economic development, environmental effects such as emissions of greenhouse gas, wasted energy, fuel consumption, and social effects such as the effects of job opportunities created. By application of a stochastic-possibilistic approach, they applied a novel hybrid meta-heuristic algorithm for solving the problem.

A fundamental study which considered perishability of products in LIRP, is that of Hiassat *et al.* (2017). The studied SC in this research was involved with multiple warehouses and retailers; the locations of warehouses, feasible routes for product distribution and retailers' inventory levels were determined. Among other researches on LIRP with perishable items, that of Tavana *et al.* (2018) can be addressed in which an integrated humanitarian LIRP was studied both in pre- and post-disaster phases. Their model determined the best locations of the warehouses, inventory management of perishable items in the pre-disaster phase, and emergency vehicles routing in the post-disaster phase. However, studies on the perishability of items in different operations management problems, specifically, inventory-centric problems, are growing such as in Gupta *et al.* (2026).

Biuki *et al.* (2020) studied LIRP in a four-layer SC with perishable items under uncertainty, taking into account the environmental effects and social responsibilities in the objectives. They developed a two-stage method for solving the problem; in the first stage, they selected the best suppliers in terms of social responsibility and environmental considerations while in the second stage, formulated the problem and solved it. Due to the difficulty of this three-objective model, two meta-heuristic hybrid algorithms based on genetic algorithm (GA) and particle swarm optimization (PSO) were developed. Zandkarimkhani *et al.* (2020) formulated a multi-product, multi-period and bi-objective LIRP for a perishable pharmaceutical item under demand uncertainty. The objectives were to minimize both the total cost and the amount of lost demands. They developed a new hybrid approach, based on fuzzy logic, FGP and chance-constrained programming to tackle the uncertainty in the model.

Another research on LIRP under uncertainty was that of Tavana *et al.* (2022) in which a multi-objective model was proposed in order to design a sustainable closed-loop SC with scenario-based uncertainty. The objectives were minimizing total costs and greenhouse gas emissions as well as maximizing job opportunities. They used an intelligent simulation algorithm to generate data and employed FGP approach to solve it. Shang *et al.* (2022) investigated LIRP in healthcare sector for the two cases of deterministic and stochastic demands. They used the robust programming for dealing with uncertainty and implemented their developed model in a real-world problem with 78 hospitals.

Nasiri *et al.* (2023) developed a new green LIRP model with simultaneous pickup and delivery. They considered demand uncertainty, storage levels of the members, time window for reaching vehicles to destinations, disruption of facilities and environmental effects in their model. They employed the scenario-based stochastic programming approach to deal with demand uncertainty and disruption risks. Navazi *et al.* (2023) developed a new green LIRP model with perishable goods, fuzzy demand and the possibility of returning goods from customers. Their model was multi-objective in which total costs and environmental impacts were minimized while productivity was maximized. The chance-constrained programming was utilized to tackle the uncertainties in the model.

Gitinavard *et al.* (2024) investigated the hub location integrated with routing problem for perishable goods warehousing. The considered objectives such as cost reduction, travel time reduction, and customer demand satisfaction with multi-commodity and multi-modal transportation systems in a dairy SC. Possibilistic robust programming was utilized to deal with the uncertainties and a new solution technique based on a fuzzy group decision-making was proposed. Zhang *et al.* (2025) proposed a two-stage model for LIRP in a relief SC considering supply, transportation times and demand to be of uncertainty. They used scenario-based optimization to tackle the addressed uncertainties in the model

2.2 Review of Decentralized LIRP

There are a considerable number of researches on the inventory decisions with at least two players in the SC; in this case, usually game-theoretic approaches specifically BLP is used to deal with these problems (Saha, 2025). However, there are a number of mixed researches of inventory and other operational management decisions such as location and routing. Wang *et al.* (2007) formulated location-inventory problem in the form of BLP in a B2C electronic SC. Location decisions were made by the leader and inventory decisions were made by the follower. Golpîra *et al.* (2017) studied design of an opportunistic SC using BLP considering environmental effects and uncertain demand tackled by scenario-based and robust programming. They used Karush–Kuhn–Tucker (KKT) conditions to solve the problem. Nourifar *et al.* (2018) studied a joint location-inventory-production-distribution problem under mixed uncertainty with two levels. The considered multi-product and multi-period SC network included factories, distribution centers and customer zones. Demand and price uncertainties were represented by stochastic and fuzzy numbers. The problem was formulated in the form of BLP in which customers were assumed at the first level and the rest of the SC as the second level. Fuzzy programming and CCP were used to tackle uncertainties in the model. Ghasemi Bijaghini and SeyedHosseini (2018) addressed a bi-level production-inventory-routing problem in a three-layer SC with decaying pharmaceutical goods in an uncertain environment. The leader aim was to reduce budgeting costs and the follower goal was to decrease the amounts of shortages. Parvasi *et al.* (2019) formulated a bi-level location-routing problem for an urban school bus system. They defined two objectives of maximizing profit and minimizing wasted time of students for the leader while for the follower, a single objective as minimizing students' expenses was considered.

Nourifar *et al.* (2020) studied a decentralized four-echelon SC including supplier, manufacturer, distributor and customer developing a tri-level programming under mixed uncertainties. They formulated an integrated location-inventory-procurement-production-distribution problem using stochastic and fuzzy tools in each layer of the chain considering uncertainties in parameters such as demand, raw materials and price. Basciftci and Van Hentenryck (2020) developed a bi-level location-routing model in which the leader was going to design a transit network and the follower selected the best route in the network. The developed model was solved utilizing a decomposition algorithm based on Benders' cuts. Korani and Eydi (2021) investigated a reliable bi-level location-routing problem in a hub network in which at the UL, the costs of designing network including the establishment of hubs, communication arcs and hub links were minimized, while at the LL, paths with the least probability of disruption or highest reliability were created. Then they used the KKT method and presented a two-phase heuristic method with a penalty function to solve the problem. Alinaghian *et al.* (2021) dealt with a green bi-level inventory-routing problem and formulated the problem by piecewise linearization method and solved it using an innovative TS algorithm.

Khanchehzarrin *et al.* (2022) presented a multi-objective bi-level model for the location-routing problem in relief SC. They considered two objectives for each level. In the first level, time and cost objectives and in the second level, efficiency and risk objectives were considered. Then, by developing KKT conditions, they converted the BLP into a single-level and solved it. The results showed that it was better to provide low-risk goods with higher priorities through public assistance to increase the efficiency of the relief SC. Achamrah *et al.* (2022) formulated a multi-product multi-period bi-level inventory-routing problem in an uncertain two-layer decentralized SC (including a manufacturer and several sales points). Both the leader and follower aimed to reduce costs including inventory and routing costs. They developed a hybrid GA combined with deep reinforcement learning to solve the problem. Valizadeh *et al.* (2023) examined the SC network of vaccine distribution during the COVID pandemic. They formulated a robust bi-level location-inventory model under scenario-based uncertain demand. In the upper level, the risk of mortality from untimely vaccine supply and the risk of inequality vaccine distribution was minimized, while the lower level considered all operational costs of vaccine distribution. They use KKT conditions to transform the BLP a single level problem and solved it for large sizes using meta-heuristic algorithm. Partovi *et al.* (2023) offered

Table 1 Summary of the Mentioned Researches and the Contribution of this Paper

Research	BLP	Uncertainly			Location	Inventory	Routing	Perishable Product
		Fuzzy	Stochastic	Robust				
Ahmadi & Azad (2010)		×	×	×		×		
Lv *et al.* (2007)				×				×
Tavana *et al.* (2018)	×	×	×	×				
Zandkarimkhani *et al.* (2020)	×	×	×	×		×	×	
Tavakkoli-Moghaddam & Raziei (2016)		×	×	×			×	
Zhalechian *et al.* (2016)		×	×	×		×	×	
Hiassat *et al.* (2017)	×	×	×	×				
Biuki *et al.* (2020)	×	×	×	×	×		×	
Navazi *et al.* (2023)	×	×	×	×			×	
Tavana *et al.* (2022)		×	×	×		×		
Shang *et al.* (2022)	×	×	×	×	×	×		
Golpîra *et al.* (2017)				×	×	×		×
Nourifar *et al.* (2018)			×	×		×	×	×
Nourifar *et al.* (2020)			×	×		×	×	×
Valizadeh *et al.* (2023)	×		×	×	×	×		×
Khanchehzarrin *et al.* (2022)		×		×				×
Wang *et al.* (2007)			×	×				×
Parvasi *et al.* (2019)		×		×				×
Basciftci & Van Hentenryck (2020)		×		×				×
Ghasemi Bijaghini & Seyed Hosseini (2018)	×	×	×		×	×		×
Alinaghian *et al.* (2021)		×	×					×
Korani & Eydi (2021)		×		×				×
Achamrah *et al.* (2022)		×	×			×		×
Pishvaee & Fazli Khalaf (2016)				×	×		×	
Dehghan *et al.* (2018)				×	×	×	×	
Rahmani & MirHassani (2015)				×				×
Farvaresh & Sepehri (2013)				×				×
Nasiri *et al.* (2023)		×	×	×		×		
Partovi *et al.* (2023)	×	×	×	×		×		×
Gitinavard *et al.* (2024)	×	×	×	×	×		×	
Zhang *et al.* (2025)		×	×	×		×		
This Paper	×	×	×	×	×	×	×	×

a novel bi-level model for the LIRP with perishable products and uncertainty in demand. They extended the fuzzy solution technique of Chen and Chen (2015) with some improvements to reduce the solution time.

The current research in this paper is an extension to Partovi *et al.* (2023) in which the bi-level LIRP is formulated using different uncertainty tools including, robust optimization, scenario-based programming and fuzzy logic. The fuzzy approach let the model consider flexibility in constraints for the two separate pessimistic and optimistic attitudes of the leader and follower. To tackle the uncertainties in the model, the robust fuzzy flexible programming proposed by Pishvaee and Fazli Khalaf (2016), Me-Measure technique proposed by Xu and Zhou (2013) and Dehghan *et al.* (2018) were utilized. As for the solution techniques, different solution techniques have been applied for solving bi-level problems such as the branch-and-bound algorithm by Bard and Moore (1990) and Farvaresh and Sepehri (2013), fuzzy set theory based solution techniques by Osman *et al.* (2004) and Chen and Chen (2015), examining KKT optimality condition for converting the linear BLP into its corresponding single-level problem by Lv *et al.* (2007) and Roghanian *et al.* (2008), and Lagrangian relaxation method by Rahmani and MirHassani (2015). For the addressed problem, the improved FGP method of Partovi *et al.* (2023) called FGP-MCGP is customized to solve the uncertain version of the problem. In this new method, the proposed FGP method of Chen and Chen (2015) is combined with the MCGP technique of Chang (2008) in one stage, instead of the initial two stages of Chen and Chen (2015). Table 1 gives a summary of the reviewed researches. It is clear from the literature that no research is done on BLLIRP under mixed uncertainty tools considering two separate pessimistic and optimistic attitudes of the leader and follower.

Table 2 Notations Used in the Mathematical Model

Variable	Definitions
J	Representing set of retailers ($j \in J$, j =1, 2, …, $\lvert J \rvert$)
P	Representing set of time periods ($p\in P$, p =1, 2, …, $\lvert P\rvert$)
P'	Representing set of time periods from the beginning to the maximum shelf-life of product. Note that we have $P \subset P'$ ($\pi\epsilon P'$, =1, 2, …, $\lvert P\rvert$, …, $\lvert P\rvert$+ π_{max}-1) (noting that π_{max}: Shelf-life of product)
SC	Representing set for scenarios ($s \in SC$, s =1, 2, …, $\lvert SC\rvert$)
FR	Representing set for feasible routes ($r\in FR$, r =1, 2, …, $\lvert FR\rvert$)
V	Representing set for homogeneous vehicles ($v \in V$, v =1, 2, …, $\lvert V\rvert$)
$\tilde{f}_i$	Trapezoidal fuzzy number of fixed opening cost of new warehouse i ($\tilde{f}_i$ =(f_{i1}, f_{i2}, f_{i3}, f_{i4}))
CV	Capacity of each vehicle
$\tilde{d}_{jp}^{s}$	Trapezoidal fuzzy number representing demand of retailer j in time period p in scenario s ($\tilde{d}_{jp}^{s}$= ($d_{jp1}{}^{s}$, $d_{jp2}{}^{s}$, $d_{jp3}{}^{s}$, $d_{jp4}{}^{s}$))
$\tilde{d}_{j\pi}^{s}$	Trapezoidal fuzzy number representing demand of retailer j in time period π in scenario s ($\tilde{d}_{j\pi}^{s}$= ($d_{j\pi1}^{s}$, $d_{j\pi2}^{s}$, $d_{j\pi3}^{s}$, $d_{j\pi4}^{s}$)). Note that for each p, $\pi \le \lvert P \rvert$ and we have: $\tilde{d}_{jp}^{s}$ =$\tilde{d}_{j\pi}^{s}$
U_{jp}^{s}	Upper bound of inventory amount of retailer j in time period p and scenario s, noting that $U_{jp}^{s} = \sum_{\pi>p}^{\pi<p+\pi_{max}} \tilde{d}_{j\pi}^{s}$
Pr_s	The probability of the occurrence of scenario s
$\tilde{h}_{jp}^{s}$	Trapezoidal fuzzy number of inventory holding cost of retailer j in time period p in scenario s (($\tilde{h}_{jp}^{s}$= (h_{jp1}^{s}, h_{jp2}^{s}, h_{jp3}^{s}, h_{jp4}^{s}))
aa_{jr}	A parameter which is equal to one if route r passes through retailer j; and zero, otherwise
$\beta\beta_{ri}$	A parameter that is equal to one if route r passes through warehouse i; and zero, otherwise
$\tilde{c}_{r}^{s}$	Trapezoidal fuzzy number representing total transportation costs of route r in scenario s ($\tilde{c}_{r}^{s}$= c_{r1}^{s}, c_{r2}^{s}, c_{r3}^{s}, c_{r4}^{s})
m_i	Binary variable which equals to one if a warehouse is opened in location i; and zero, otherwise
θ_{rp}^{s}	Binary variable which equals to one if route r is selected in time period p and scenario s; and zero, otherwise
I_{jp}^{s}	The amount of inventory of retailer j at the end of time period p in scenario s
Q_{jrp}^{s}	The quantity of product received by retailer j in route r in time period p in scenario s

An important feature of the proposed method is that it integrates the addressed approaches jointly rather than applying them sequentially. In other words, this research uses a scenario approach to deal with uncertainty in model parameters that are predictable based on historical data and uses the fuzzy flexible uncertainty approach to deal with the uncertainty in the model's inequalities or constraints, and finally, the robust optimization approach is provided for the giving reliable solutions for the model. In previous research, the robust flexible approach has been proposed and used by Pishvaee and Fazli Khalaf (2016). Also, the robust scenario approach was presented by Mulvey *et al.* (1995). In this research, the features and advantages of both addressed studies integrated in order to propose a robust flexible stochastic model.

3. METHODOLOGY

First, we describe the problem and notations and then present the models in this section.

3.1 Description of the Problem and Notations

The developed model for the BLLIRP in this research is for a two-echelon SC consisting of several distribution centers/warehouses in the first echelon and several retailers in the second echelon. A perishable product with specific storage life (or shelf-life) is assumed to be sent from the warehouses to retailers. The shelf-life means that after a known period of time, the product will be spoiled. According to Hiassat *et al.* (2017), the inventory level at each retailer is determined based on the product's shelf-life rather than the space capacity. Three scenarios for the uncertainty conditions including pessimistic, moderate and optimistic economic conditions are considered.

The demand of retailers and all operational costs are considered to be uncertain. In addition, we assume that there are some flexible constraints in the proposed model. There are feasible routes (called tours) that start from a specific warehouse and end at the same warehouse after meeting a number of retailers. It is assumed that homogeneous vehicles are used to distribute the products through the tours. Naturally, the capacity of each vehicle should be greater than the maximum demand of retailers in each time period. Additionally, each retailer can be visited at most once in each tour. According to the game structure of the problem, warehouses are managed by a central management called UL, and retailers are managed by another central management called LL, as shown in Figure 1. The notations are as in Table 2.

Before giving the different steps for making the final robust flexible stochastic mathematical model, we show sequential stages of this research as in Figure 2.

3.2 Development of Flexible Stochastic-fuzzy BLLIRP Model

The initially proposed uncertain bi-level model in Equations (1)-(13) is called flexible stochastic-fuzzy BLLIRP since it is a scenario-based model with fuzzy parameters. It should be noted that, the symbol " ˜ " on top of notations represents fuzziness.

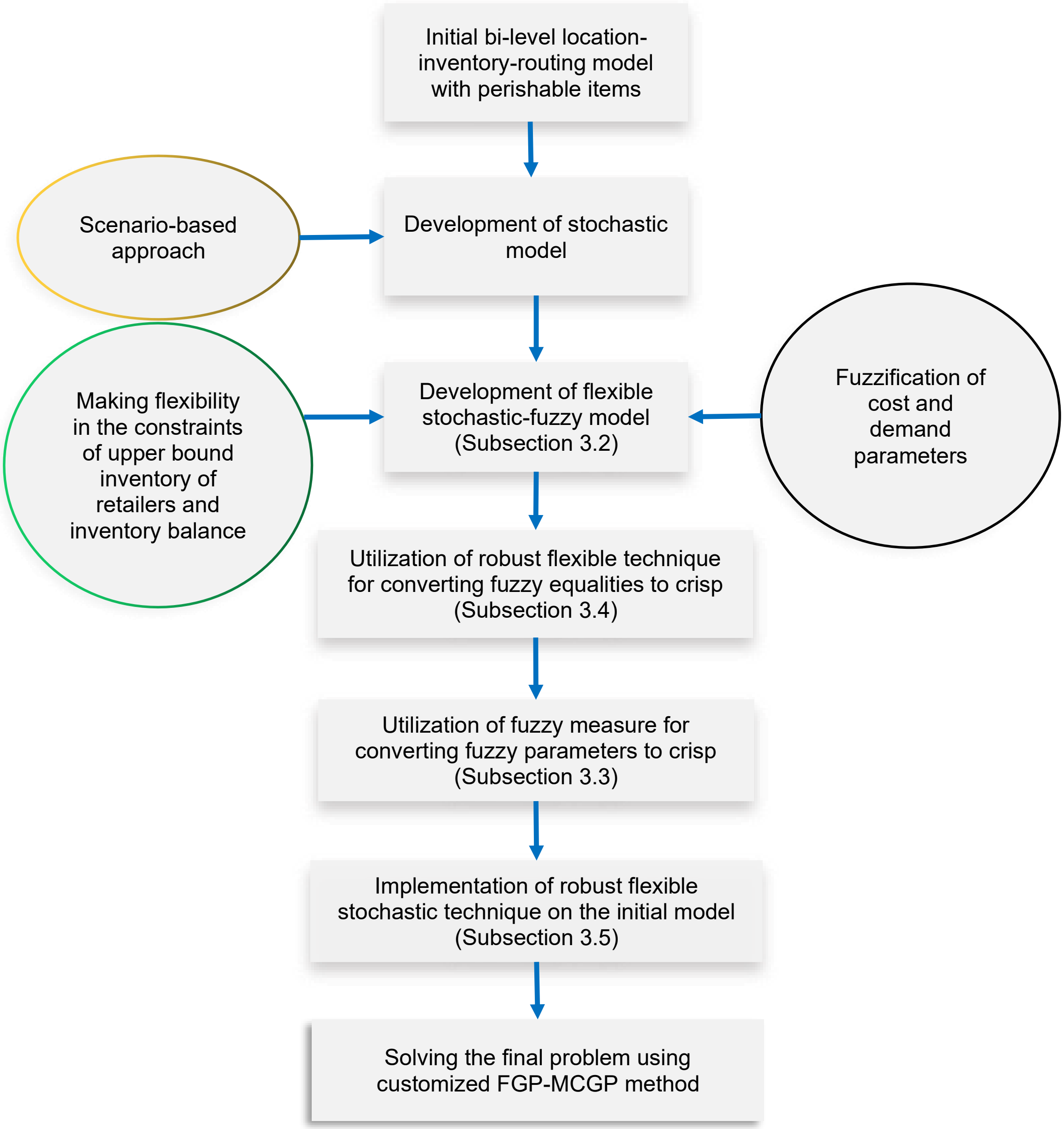

Figure 2 Research Steps Including Modeling and Solving

Leader Model:

$$Min\ Tc_1 = \sum_{i \in I} \tilde{f}_i\, m_i + \sum_{s \in SC} \sum_{p \in P} \sum_{r \in FR} Pr_s\, \tilde{c}_r^s\, \theta_{rp}^s \quad (1)$$

subject to:

$$\sum_{r \in FR} aa_{jr}\, \theta_{rp}^s \leq 1, \forall j \in J, \forall p \in P, \forall s \in SC \quad (2)$$

$$\theta_{rp}^s \leq \sum_{i \in I} \beta\beta_{ri} m_i\, , \forall r \in FR, \forall p \in P, \forall s \in SC \quad (3)$$

$$\sum_{r \in FR} \theta_{rp}^s \leq |V|, \forall p \in P, \forall s \in SC \quad (4)$$

$$\sum_{r \in FR} \theta_{rp}^s \geq 1, \forall p \in P, \forall s \in SC \quad (5)$$

$$m_i, \theta_{rp}^s \in \{0,1\}, \forall p \in P, \forall i \in I, \forall r \in FR, \forall s \in SC \quad (6)$$

Follower Model:

$$Min\ TC_2 = \sum_{s \in SC} \sum_{p \in P} \sum_{j \in J} Pr_s\, \tilde{h}_{jp}^s\, I_{jp}^s \quad (7)$$

subject to:

$$I_{jp-1}^s + \sum_{r \in FR} aa_{jr}\, Q_{jrp}^s \cong \tilde{d}_{jp}^s + I_{jp}^s, \forall p \in P, \forall j \in J, \forall s \in SC \quad (8)$$

$$U_{jp}^s \cong \sum_{\pi > p}^{\pi < p + \pi_{max}} \tilde{d}_{j\pi}^s\, , \forall p \in P\, , \forall j \in J, \forall s \in SC \quad (9)$$

$$I_{jp}^s \leq U_{jp}^s, \forall p \in P, \forall j \in J, \forall s \in SC \quad (10)$$

$$\sum_{j\in J} Q^{s}_{jrp} \le CV\theta^{s}_{rp}, \forall p\in P, \forall r\in FR, \forall s\in SC \tag{11}$$

$$I^{s}_{jp} \ge 0, \forall j\in J, \forall p\in P, \forall s\in SC \tag{12}$$

$$Q^{s}_{jrp} \ge 0, \forall j\in J, \forall r\in FR, \forall p\in P, \forall s\in SC \tag{13}$$

As for the leader (or UL), the given objective function in Equation (1) represents the sum of costs of opening warehouses and product transportation to the retailers. Constraint (2) ensures that each retailer is visited in every time period and scenario. Constraint (3) guarantees that every tour starts from and ends with an open warehouse in each period and scenario. Constraint (4) limits the number of used tours to less than or equal to the number of available vehicles in each time period and scenario. Constraint (5) guarantees that at least one rout must be used in each time period and scenario. Constraint (6) indicates the status of the two decision variables of the leader.

As for the follower, the objective function presented in (7) represents the total inventory holding costs at retailers under all scenarios. Inventory balance equation is given in Constraints (8). According to this constraint, the sum of initial inventory level and the amount of goods received at each retailer in each time period is equal to the retailer's demand and the final inventory level in the same period in each scenario. Constraint (9) calculates the upper bound inventory level of each retailer in each time period and in each scenario. It should be noted that the upper bound inventory level should satisfy the total demand during the shelf-life of the product, i.e., $p+\pi_{max}-1$. We have assumed Constraint (9) to be flexible which means that there can be a small flexibility in its fully satisfaction. Constraint (10) ensures that each retailer's inventory can't exceed its upper bound. Constraints (11) controls the used vehicle capacity. Constraints (12) and (13) give the status of the decision variables of the follower.

In the rest of the paper, in Sub-section 3-3, the fuzzy measures and the optimistic-pessimistic attitudes of DMs and how to use it in the modeling, are expressed. Then, in Sub-section 3-4, the formulating of the robust model with respect to the uncertain parameters and flexible constraints, are presented. This section shows that how such an uncertain model will be eventually converted into a robust flexible stochastic model considering optimistic-pessimistic attitudes of DMs. Finally, in Sub-section 3-5, the final mathematical uncertain model is formulated in Equations (28)-(43).

3.3 Fuzzy Measures

The most famous fuzzy measures are possibility (Pos), necessity (Nec), credibility (Cr) and fuzzy general measure (Me) (Torabi and Towfighi, 2017). Pos represents the most possible level of a phenomenon with optimistic attitude, Nec represents the most pessimistic level of a phenomenon, while Cr represents the average of the two possibility and necessity measures. Xu and Zhou (2013) offered a novel fuzzy measure called general fuzzy measure shown by Me which is the general model of Cr measure. In this measure, the DM can utilize a pessimistic-optimistic parameter, shown by λ for considering a convex mixture of optimistic or pessimistic spectrum. Indeed, the Me measure allows the DM to adopt a hybrid attitude (Dehghan *et al.,* 2018; Xu and Zhou, 2013).

The Me measure is defined as in Equation (14):

$$Me\,(A) = Nec(A) + \lambda[Pos(A) - Nec(A)] = \lambda Pos(A) + (1-\lambda)Nec(A) \tag{14}$$

Considering the above equation, we can conclude that: If $\lambda = 0$, then *Me* = *Nec* which means considering the least chance for the occurrence of a phenomenon. If $\lambda = 0.5$, then *Me* = *Cr* which means a middle chance for its occurrence and if $\lambda = 1$, then *Me* = *Pos* which means the highest chance for its occurrence. In fact, when λ takes higher values, it is more possible having higher values for a fuzzy number and if the fuzzy number is of cost type, then it means that the DM's attitude is of pessimistic type.

Considering a trapezoidal fuzzy variable $\widetilde{K}$ as $\widetilde{K} = (k_1, k_2, k_3, k_4)$, in which $k_1 \le k_2 \le k_3 \le k_4$, its membership function can be given as in Equation (15) (Xu and Zhou, 2013):

$$\mu_{\widetilde{K}}(y) = \begin{cases} \frac{y-k_1}{k_2-k_1} & \text{if } k_1 \le y \le k_2 \\ 1 & \text{if } k_2 \le y \le k_3 \\ \frac{k_4-y}{k_4-k_3} & \text{if } k_3 \le y \le k_4 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

The general *Me* measures of $\widetilde{K} \le y$, $\widetilde{K} \ge y$ can be obtained from Equations (16) and (17) (Xu and Zhou, 2013):

$$Me(\widetilde{K} \le y) = \begin{cases} 0 & \text{if } y \le k_1 \\ \lambda \dfrac{y-k_1}{k_2-k_1} & \text{if } k_1 \le y \le k_2 \\ \lambda & \text{if } k_2 \le y \le k_3 \\ \lambda + (1-\lambda)\dfrac{y-k_3}{k_4-k_3} & \text{if } k_3 \le y \le k_4 \\ 1 & \text{if } k_4 \le y \end{cases} \tag{16}$$

$$Me(\widetilde{K} \ge y) = \begin{cases} 1 & \text{if } y \le k_1 \\ \lambda + (1-\lambda)\dfrac{k_2-y}{k_2-k_1} & \text{if } k_1 \le y \le k_2 \\ \lambda & \text{if } k_2 \le y \le k_3 \\ \lambda \dfrac{k_4-y}{k_4-k_3} & \text{if } k_3 \le y \le k_4 \\ 0 & \text{if } k_4 \le y \end{cases} \tag{17}$$

The expected value of $\widetilde{K} = (k_1, k_2, k_3, k_4)$ based on *Me* measure can be defined as in Equation (18) (Xu and Zhou, 2013):

$$E^{Me}[\widetilde{K}] = \left(\frac{1-\lambda}{2}\right)(k_1 + k_2) + \frac{\lambda}{2}(k_3 + k_4) \tag{18}$$

If the confidence level is $\delta > 0.5$, we will have the following propositions based on Me measure in Equations (19) and (20) for the trapezoidal fuzzy number $\widetilde{K} = (k_1, k_2, k_3, k_4)$ (Dehghan *et al.,* 2018):

$$Me\{\widetilde{K} \ge y\} \ge \delta \;\Leftrightarrow\; y \le \frac{(\delta-\lambda)k_1 + (1-\delta)k_2}{1-\lambda} \tag{19}$$

$$Me\{\widetilde{K} \le y\} \ge \delta \;\Leftrightarrow\; y \ge \frac{(\delta-\lambda)k_4 + (1-\delta)k_3}{1-\lambda} \tag{20}$$

3.4 Development of Robust Flexible Stochastic Model

Consider the compact general form of a fuzzy mathematical model in Equations (21) in which y is a binary variable and x_s represents continuous variables of the model. The parameters of the model are $\tilde{c}_s$, $\tilde{f}$ and $\tilde{d}_s$ while the coefficient matrices are shown by A, B, S and $\tilde{N}$. Vectors $\tilde{c}_s$ and $\tilde{d}_s$ represent the scenario-based trapezoidal fuzzy numbers of costs (including transportation cost, fixed opening cost of warehouse, and inventory holding cost) and demand, respectively.

$$Min\ Z = \tilde{f}y + \tilde{c}_s x_s$$

$$\begin{aligned} &subject\ to: \\ &Ax_s \geq \tilde{d}_s, \forall s \\ &Bx_s = 0, \forall s \\ &Sx_s \leq \tilde{N}y, \forall s \\ &Ty \leq 1 \\ &y \in \{0,1\}, x_s \geq 0, \forall s \end{aligned} \tag{21}$$

According to the *Me* measure equations in (19) and (20), and the flexible constraint approach in Pishvaee and Fazli Khalaf (2016), the basic flexible stochastic-fuzzy model can be given as in Equation (22) in which δ_s and ρ_s ($0 \leq \delta_s$ and $\rho_s \leq 1$) are the least confidence levels for the flexible constraints of the model.

$$Min\ E(z) = E[\tilde{f}]y + E[\tilde{c}_s]x_s$$

$$\begin{aligned} &subject\ to: \\ &Me\{Ax_s \gtrapprox \tilde{d}_s\} \geq \delta_s,\ \forall s \\ &Bx_s = 0,\ \forall s \\ &Me\{Sx_s \lessapprox \tilde{N}y\} \geq \delta_s, \forall s \\ &Ty \leq 1 \\ &y \in \{0,1\}, x_s \geq 0, \forall s \end{aligned} \tag{22}$$

According to Pishvaee and Fazli Khalaf (2016), we show the deviations in the two flexible constraints by two trapezoidal fuzzy numbers $\tilde{t} = (t_1, t_2, t_3, t_4)$ and $\tilde{K} = (k_1, k_2, k_3, k_4)$. Representing the confidence levels for the flexible constraints of the model by α_s and β_s ($0 \leq \alpha_s, \beta_s \leq 1$), and the confidence levels for chance constraints by δ_s and ρ_s as mentioned earlier, then, the crisp equivalent of the model (22) can be given as in (23) considering that the probability of each scenario s is shown by Pr_s. It should be noted that λ is a pessimistic-optimistic parameter for considering a convex mixture of optimistic or pessimistic spectrum of DM's attitude to risk.

$$\begin{aligned} &min\ \ E(z) = \left[\frac{(1-\lambda)}{2}(f_1 + f_2) + \frac{\lambda}{2}(f_3 + f_4)\right]y \\ &+ \sum_s Pr_s \left[\frac{(1-\lambda)}{2}(c_1^s + c_2^s) + \frac{\lambda}{2}(c_3^s + c_4^s)\right]x_s \\ &subject\ to: \\ &Ax_s \geq \frac{(\delta_s - \lambda)d_4^s + (1-\delta_s)d_3^s}{1-\lambda} \\ &- \left[\left[\frac{1-\lambda}{2}(t_1^s + t_2^s) + \frac{\lambda}{2}(t_3^s + t_4^s)\right](1-\alpha_s)\right],\ \forall s \\ &Bx_s = 0, \quad \forall s \\ &Sx_s \leq \left[\frac{(\rho_s - \lambda)N_1^s + (1-\rho_s)N_2^s}{1-\lambda}\right]y \\ &+ \left[\left[\frac{1-\lambda}{2}(k_1^s + k_2^s) + \frac{\lambda}{2}(k_3^s + k_4^s)\right](1-\beta_s)\right]y, \quad \forall s \\ &Ty \leq 1 \\ &y \in \{0,1\}, x_s \geq 0,\ \ 0.5 \leq \delta_s, \rho_s \leq 1,\ \ 0 \leq \alpha_s, \beta_s \leq 1, \forall s \end{aligned} \tag{23}$$

In model (24), we have shown the robust counterpart of the problem, called robust flexible stochastic model, in order to find a feasible solution that has minimal deviations from the optimal solution.

$$\begin{aligned} &Min\ \ E(z) + \psi[Z_{max} - E(Z)] + \eta \sum_s Pr_s . |E(z) - E(z_s)| \\ &+\omega \sum_s Pr_s \left[d_4^s - \left(\frac{(\delta_s - \lambda)d_4^s + (1-\delta_s)d_3^s}{1-\lambda}\right)\right] + \\ &\vartheta \sum_s Pr_s \left[\left(\frac{(\rho_s - \lambda)N_1^s + (1-\rho_s)N_2^s}{1-\lambda}\right) - N_1^s\right]y + \\ &\gamma \sum_s Pr_s \left[\left[\frac{1-\lambda}{2}(t_1^s + t_2^s) + \frac{\lambda}{2}(t_3^s + t_4^s)\right](1-\alpha_s)\right] + \\ &\Theta \sum_s Pr_s \left[\left[\frac{1-\lambda}{2}(k_1^s + k_2^s) + \frac{\lambda}{2}(k_3^s + k_4^s)\right](1-\beta_s)\right]y \\ &= \left[\frac{1-\lambda}{2}(f_1 + f_2) + \frac{\lambda}{2}(f_3 + f_4)\right]y \\ &+ \sum_s Pr_s \left[\frac{1-\lambda}{2}(c_1^s + c_2^s) + \frac{\lambda}{2}(c_3^s + c_4^s)\right]x_s \\ &+\psi \left[\begin{array}{c} \left(f_4 y + \sum_s P_s c_4^s x_s\right) \\ - \left[\begin{array}{l} \left[\frac{1-\lambda}{2}(f_1 + f_2) + \frac{\lambda}{2}(f_3 + f_4)\right]y + \\ \sum_s Pr_s \left[\frac{1-\lambda}{2}(c_1^s + c_2^s) + \frac{\lambda}{2}(c_3^s + c_4^s)\right]x_s \end{array}\right] \end{array}\right] \\ &+\eta \sum_s Pr_s . \left| \begin{array}{c} \left\{\begin{array}{l} \left[\frac{1-\lambda}{2}(f_1 + f_2) + \frac{\lambda}{2}(f_3 + f_4)\right]y + \\ \sum_s Pr_s \left[\frac{1-\lambda}{2}(c_1^s + c_2^s) + \frac{\lambda}{2}(c_3^s + c_4^s)\right]x_s \end{array}\right\} \\ - \left\{\begin{array}{l} \left[\frac{1-\lambda}{2}(f_1 + f_2) + \frac{\lambda}{2}(f_3 + f_4)\right]y \\ + \left[\frac{1-\lambda}{2}(c_1^s + c_2^s) + \frac{\lambda}{2}(c_3^s + c_4^s)\right]x_s \end{array}\right\} \end{array} \right| \\ &+\omega \sum_s Pr_s \left[d_4^s - \left(\frac{(\delta_s - \lambda)d_4^s + (1-\delta_s)d_3^s}{1-\lambda}\right)\right] + \\ &\vartheta \sum_s Pr_s \left[\left(\frac{(\rho_s - \lambda)N_1^s + (1-\rho_s)N_2^s}{1-\lambda}\right) - N_1^s\right]y \\ &+\gamma \sum_s Pr_s \left[\left[\frac{1-\lambda}{2}(t_1^s + t_2^s) + \frac{\lambda}{2}(t_3^s + t_4^s)\right](1-\alpha_s)\right] + \\ &\Theta \textstyle\sum_s Pr_s \left[\left[\frac{1-\lambda}{2}(k_1^s + k_2^s) + \frac{\lambda}{2}(k_3^s + k_4^s)\right](1-\beta_s)\right]y \end{aligned}$$

$$\begin{aligned} &subject\ to: \\ &Ax_s \geq \left(\frac{(\delta_s - \lambda)d_4^s + (1-\delta_s)d_3^s}{1-\lambda}\right) \\ &- \left[\left[\frac{1-\lambda}{2}(t_1^s + t_2^s) + \frac{\lambda}{2}(t_3^s + t_4^s)\right](1-\alpha_s)\right] \quad ,\forall s \end{aligned}$$

$$Bx_s = 0 \quad ,\forall s$$
$$Sx_s \le \left(\frac{(\rho_s-\lambda)N_1^s + (1-\rho_s)N_2^s}{1-\lambda}\right)y + \left[\left[\frac{1-\lambda}{2}(k_1^s+k_2^s) + \frac{\lambda}{2}(k_3^s+k_4^s)\right](1-\beta_s)\right]y \quad ,\forall s \tag{24}$$
$$Ty \le 1$$
$$y \in \{0,1\}, x_s \ge 0, 0 \le \alpha_s, \beta_s \le 1, 0.5 \le \delta_s, \rho_s \le 1 \quad ,\forall s$$

In the robust flexible stochastic model given in (24), the first term in the objective function calculates the expected value of z. The second term represents the optimality robustness, which can be controlled via minimizing the maximum possible value (i.e., Z_{max}) and the expected value (i.e., $E(z)$). Indeed, the second term with weight of ψ is employed to control the optimality robustness under fuzzy parameters and is called possibilistic deviation. It should be noted that Z_{max} and $E(z)$) are obtained from Equations (25) and (26) (Pishvaee and Fazli Khalaf, 2016; Pishvaee *et al.,* 2012):

$$Z_{max} = f_4 y + \sum_s Pr_s \,.\, c_4^s x_s \tag{25}$$

$$E(z) = \left[\frac{1-\lambda}{2}(f_1+f_2) + \frac{\lambda}{2}(f_3+f_4)\right]y + \sum_s \Pr_s \left[\frac{1-\lambda}{2}(c_1^s+c_2^s) + \frac{\lambda}{2}(c_3^s+c_4^s)\right]x_s \tag{26}$$

The third term with weight of η corresponds to the scenario-based uncertainties called scenario-based deviation from the optimal value in the objective function. This term reduces the amount of violation of E(z) from expected value of the objective function for each scenario, i.e., $E(z_s)$ obtained from Equations (27).

$$E(z_s) = \left[\frac{1-\lambda}{2}(f_1+f_2) + \frac{\lambda}{2}(f_3+f_4)\right]y + \left[\frac{1-\lambda}{2}(c_1^s+c_2^s) + \frac{\lambda}{2}(c_3^s+c_4^s)\right]x_s \tag{27}$$

Furthermore, in the objective function, the fourth and fifth terms correspond to the feasibility robustness related to the chance constraints with weights of ω and ϑ. Similarly, the sixth and seventh terms correspond to the feasibility robustness related to the flexible constraints with weights of γ and Θ. Note that in model (24), we employed the *Me* measure with parameter λ to formulate the pessimistic-optimistic attitude of the DM into the problem. The compact robust model presented in Equation (24) has one DM and therefore we only use single parameters of λ in the model.

3.5 Implementing the Robust Flexible Stochastic Model on BLLIRP Model

The proposed general robust flexible stochastic given in (24), can be embedded into the flexible stochastic fuzzy BLLIRP model given in Equations (1)-(13). The obtained model is given as in Equations (28)-(43). The fixed cost of opening each warehouse is assumed to be a fuzzy number and independent from different scenarios while demand and the rest of the cost parameters are assumed to be fuzzy and scenario-based as in Dehghan et al., 2018. Another point in the model is on using *Me* measure. Since in the addressed model, there are two DMs in the UL and LL, we need to consider two pessimistic and optimistic modes for parameter of λ, denoted by λ_1 and λ_2 for the UL and LL attitudes, respectively.

All notations in the model are the same as those introduced in Section 3.1; in addition, the following new notations are added:

$\tilde{t}_s$: The amount of violation for the flexible constraints of the model shown by trapezoidal fuzzy numbers for each scenario s,

$\tilde{k}_s$: The amount of violation for the flexible constraints of the model shown by trapezoidal fuzzy number for each scenario s, $\tilde{k}_s = (k_1^s, k_2^s, k_3^s, k_4^s)$

ψ_1: The weight (importance) of the possibilistic deviation against other terms in the leader's objective function

η_1: The weight (importance) of the stochastic deviation against other terms in the leader's objective function

ψ_2: The weight (importance) of the possibilistic deviation against other terms in the follower's objective function

η_2: The weight (importance) of the stochastic deviation against other terms in the follower's objective function

ω: Penalty rate for each violation in constraints due to the existence of uncertain parameters

ϑ: Penalty rate for each violation in constraints due to the existence of uncertain parameters

γ: Penalty rate for exceeding the right-hand side value of flexible constraints

Θ: Penalty rate for exceeding the right-hand side value of flexible constraints

δ_s: Minimum confidence level of uncertain parameters of scenario s

α_s: Minimum confidence level corresponding to flexible constraints of scenario s

β_s: Minimum confidence level corresponding to flexible constraints of scenario s

λ_1: The UL pessimistic-optimistic attitude in Me measure approach $(0 \le \lambda_1 \le 1)$

λ_2: The LL pessimistic-optimistic attitude in Me measure approach $(0 \le \lambda_2 \prec 1)$

The robust flexible stochastic BLLIRP model can be given as in Equations (28)-(43):

UL:

$$Min\, TC_1 = \sum_{i\in I}\left[\frac{1-\lambda_1}{2}(f_{i1}+f_{i2}) + \frac{\lambda_1}{2}(f_{i3}+f_{i4})\right]m_i + \sum_{s\in SC}\sum_{p\in P}\sum_{r\in FR} Pr_s\left[\frac{1-\lambda_1}{2}(c_{r1}^s+c_{r2}^s) + \frac{\lambda_1}{2}(c_{r3}^s+c_{r4}^s)\right]\theta_{rp}^s +$$

$$+\psi_1\left[\begin{array}{l}(\sum_{i\in I} f_{i4}m_i + \sum_{s\in SC}\sum_{p\in P}\sum_{r\in FR} Pr_s c_{r4}^s \theta_{rp}^s) - \\ (\sum_{i\in I}\left[\frac{1-\lambda_1}{2}(f_{i1}+f_{i2}) + \frac{\lambda_1}{2}(f_{i3}+f_{i4})\right]m_i + \\ \sum_{s\in SC}\sum_{p\in P}\sum_{r\in FR} Pr_s\left[\frac{1-\lambda_1}{2}(c_{r1}^s+c_{r2}^s) + \frac{\lambda_1}{2}(c_{r3}^s+c_{r4}^s)\right]\theta_{rp}^s)\end{array}\right]$$

$$+\eta_1 \sum_{s\in SC} Pr_s.\left\{\begin{array}{l}\left\{\begin{array}{l}\sum_{i\in I}\begin{bmatrix}\frac{1-\lambda_1}{2}(f_{i1}+f_{i2})\\+\frac{\lambda_1}{2}(f_{i3}+f_{i4})\end{bmatrix}m_i+\\ \sum_{s\in SC}\sum_{p\in P}\sum_{r\in FR} Pr_s\begin{bmatrix}\frac{1-\lambda_1}{2}(c^s_{r1}+c^s_{r2})\\+\frac{\lambda_1}{2}(c^s_{r3}+c^s_{r4})\end{bmatrix}\theta^s_{rp}+\end{array}\right\}-\\ \left\{\begin{array}{l}\sum_{i\in I}\begin{bmatrix}\frac{1-\lambda_1}{2}(f_{i1}+f_{i2})\\+\frac{\lambda_1}{2}(f_{i3}+f_{i4})\end{bmatrix}m_i+\\ \sum_{p\in P}\sum_{r\in FR}\begin{bmatrix}\frac{1-\lambda_1}{2}(c^s_{r1}+c^s_{r2})\\+\frac{\lambda_1}{2}(c^s_{r3}+c^s_{r4})\end{bmatrix}\theta^s_{rp}+\end{array}\right\}\end{array}\right\} \tag{28}$$

subject to:

$$\sum_{r\in FR} aa_{jr}\,\theta^s_{rp} \le 1, \forall j\in J, \forall p\in P, \forall s\in SC \tag{29}$$

$$\theta^s_{rp} \le \sum_{i\in I} \beta\beta_{ri} m_i\,, \forall r\in FR, \forall p\in P, \forall s\in SC \tag{30}$$

$$\sum_{r\in FR}\theta^s_{rp} \le |V|, \forall p\in P, \forall s\in SC \tag{31}$$

$$\sum_{r\in FR}\theta^s_{rp} \ge 1, \forall p\in P, \forall s\in SC \tag{32}$$

$$m_i, \theta^s_{rp} \in \{0,1\}, \forall p\in P, \forall i\in I, \forall r\in FR, \forall s\in SC \tag{33}$$

LL:

$$MinTc_2 = \sum_{s\in SC}\sum_{p\in P}\sum_{j\in J} Pr_s\begin{bmatrix}\frac{1-\lambda_2}{2}\left(h^s_{jp1}+h^s_{jp2}\right)\\+\frac{\lambda_2}{2}\left(h^s_{jp3}+h^s_{jp4}\right)\end{bmatrix} I^s_{jp}$$

$$+\psi_2\left[\begin{array}{l}\left(\sum_{s\in SC}\sum_{p\in P}\sum_{j\in J} Pr_s h^s_{jp4} I^s_{jp}\right)-\\ \left(\sum_{s\in SC}\sum_{p\in P}\sum_{j\in J} Pr_s\begin{bmatrix}\frac{1-\lambda_2}{2}\left(h^s_{jp1}+h^s_{jp2}\right)\\+\frac{\lambda_2}{2}\left(h^s_{jp3}+h^s_{jp4}\right)\end{bmatrix} I^s_{jp}\right)\end{array}\right]$$

$$+\eta_2 \sum_{s\in SC} Pr_s.\left[\begin{array}{l}\sum_{s\in SC}\sum_{p\in P}\sum_{j\in J} Pr_s\begin{bmatrix}\frac{1-\lambda_2}{2}\left(h^s_{jp1}+h^s_{jp2}\right)\\+\frac{\lambda_2}{2}\left(h^s_{jp3}+h^s_{jp4}\right)\end{bmatrix} I^s_{jp}-\\ \sum_{p\in P}\sum_{j\in J}\begin{bmatrix}\frac{1-\lambda_2}{2}\left(h^s_{jp1}+h^s_{jp2}\right)\\+\frac{\lambda_2}{2}\left(h^s_{jp3}+h^s_{jp4}\right)\end{bmatrix} I^s_{jp}\end{array}\right]$$

$$+\omega\left[\sum_{s\in SC}\sum_{\pi\in P'}\sum_{j\in J} Pr_s\left(d^s_{j\pi4}-\left(\frac{\begin{array}{c}(\delta_s-\lambda_2)d^s_{j\pi4}\\+(1-\delta_s)d^s_{j\pi3}\end{array}}{1-\lambda_2}\right)\right)\right]+$$

$$\vartheta\sum_{s\in SC}\sum_{\pi\in P'}\sum_{j\in J} Pr_s\left(\frac{(\delta_s-\lambda_2)d^s_{j\pi1}+(1-\delta_s)d^s_{j\pi2}}{1-\lambda_2}-d^s_{j\pi1}\right)$$

$$+\gamma\sum_{s\in SC} Pr_s\left[\left[\frac{1-\lambda_2}{2}(t^s_1+t^s_2)+\frac{\lambda_2}{2}(t^s_3+t^s_4)\right](1-\alpha_s)\right]+$$

$$\Theta\sum_{s\in SC} Pr_s\left[\left[\frac{1-\lambda_2}{2}(k^s_1+k^s_2)+\frac{\lambda_2}{2}(k^s_3+k^s_4)\right](1-\beta_s)\right] \tag{34}$$

subject to:

$$I^s_{jp-1}+\sum_{r\in FR} aa_{jr}\,Q^s_{jrp} \ge \left(\frac{(\delta_s-\lambda_2)d^s_{jp4}+(1-\delta_s)d^s_{jp3}}{1-\lambda_2}\right) +I^s_{jp}, \forall p\in P, \forall j\in J, \forall s\in SC \tag{35}$$

$$I^s_{jp-1}+\sum_{r\in FR} aa_{jr}\,Q^s_{jrp} \le \left(\frac{(\delta_s-\lambda_2)d^s_{jp1}+(1-\delta_s)d^s_{jp2}}{1-\lambda_2}\right) +I^s_{jp}, \forall p\in P, \forall j\in J, \forall s\in SC \tag{36}$$

$$U^s_{jp} \ge \sum_{\pi>p}^{\pi<p+\pi_{max}}\left(\frac{(\delta_s-\lambda_2)d^s_{j\pi4}+(1-\delta_s)d^s_{j\pi3}}{1-\lambda_2}\right) -\left[\frac{1-\lambda_2}{2}(t^s_1+t^s_2)+\frac{\lambda_2}{2}(t^s_3+t^s_4)\right](1-\alpha_s) ,\forall p\in P, \forall j\in J, \forall s\in SC \tag{37}$$

$$U^s_{jp} \le \sum_{\pi>p}^{\pi<p+\pi_{max}}\left(\frac{(\delta_s-\lambda_2)d^s_{j\pi1}+(1-\delta_s)d^s_{j\pi2}}{1-\lambda_2}\right) +\left[\frac{1-\lambda_2}{2}(k^s_1+k^s_2)+\frac{\lambda_2}{2}(k^s_3+k^s_4)\right](1-\beta_s) ,\forall p\in P, \forall j\in J, \forall s\in SC \tag{38}$$

$$I^s_{jp} \le U^s_{jp}, \forall p\in P, \forall j\in J, \forall s\in SC \tag{39}$$

$$\sum_{j\in J} Q^s_{jrp} \le CV\theta^s_{rp}, \forall p\in P, \forall r\in FR, \forall s\in SC \tag{40}$$

$$I^s_{jp} \ge 0, \forall j\in J, \forall p\in P, \forall s\in SC \tag{41}$$

$$Q^s_{jrp} \ge 0, \forall j\in J, \forall r\in FR, \forall p\in P \tag{42}$$

$$0 \le \alpha_s, \beta_s \le 1, 0.5 \le \delta_s \le 1 \quad ,\forall s\in \mathrm{SC} \tag{43}$$

In the UL model, the first term in the objective function in Equation (28) represents the total expected costs, where the first part represents the cost of opening central warehouses, while the second part represents the transportation costs for delivery of goods from warehouses to retailers via the selected tours under all possible scenarios. The second term in the objective function (28), represents the optimality robustness under fuzzy parameters with weight of ψ_1 (i.e., possible deviation). The third term in the objective function (28) with weight of η_1 deals with optimality robustness under scenario-based uncertainties (i.e., stochastic deviation). Constraints (29)-(33) are as the same as Constraints (2)-(6), elaborated earlier in Section 3.2.

In the LL model, the term part in the objective function in Equation (34) represents the total expected inventory holding costs in retailers under all possible scenarios. The second term in the objective function with weight of ψ_2 represents the optimality robustness under fuzzy-based parameters (i.e., possible deviation) while the third term with weight of η_2 represents the optimality robustness under scenario-based parameters (i.e., stochastic deviation). In addition, in the objective function (34), the fourth, fifth, sixth and seventh terms deal with the feasibility robustness, where ω and ϑ are penalty rates for each unit of infeasibility of the constraints due to the inaccuracy of the uncertain parameters in chance constraints. Similarly, γ and Θ are penalty rates for violation in the right-hand side value of the flexible constraints.

The inventory balance equations are given in Constraints (35) and (36). Constraints (37) and (38) calculate the upper bound of inventory level for the retailers in each time period and scenario. In fact, the Constraints (37) and (38) are assumed to be soft or flexible. This assumption in the real-world applications means that when it's necessary to have more storage capacity than upper bound inventory, the needed space can be provided by paying more charges. Constraints (39)-(42) are the same as Constraints (10)-(12), which were explained earlier in Section 3.2. Constraint (43) expresses the acceptable intervals for the minimum confidence levels in uncertain parameters and soft constraints.

It should be noted that the terms associated with feasibility robustness only exist in the follower's objective function while the leader only has optimality robustness terms in its objective function. This is because in the leader's constraints, there are no fuzzy parameters or soft fuzzy constraint which require the feasibility robustness terms in the leader 's objective function. The terms related to optimality robustness are shown by the coefficients of ψ_1 and η_1 in the leader 's objective function while for the follower, except for the optimality robustness terms, there are two terms pertinent to feasibility robustness whose coefficients are represented by ω and ϑ

4. SOLUTION TECHNIQUES

As discussed earlier, there are different techniques to solve the addressed problems. The two-stage FGP solution technique presented by Chen and Chen (2015) can be named as one of the rather recent techniques. Considering the shortcomings and complexities of the FGP method, revisions have been made in earlier researches in order to make faster its implementation. Partovi *et al.* (2023) made revisions to FGP method of Chen and Chen (2015), employing the MCGP technique of Chang (2008) in the FGP method and called it FGP-MCGP, which resulted in better solutions in shorter times. We have customized the FGP-MCGP method offered by Partovi *et al.* (2023) in this research to solve the robust flexible stochastic problem.

Before elaborating how to implement the FGP-MCGP technique, it's useful to briefly describe the FGP method. There are two stages in the FGP; the first stage is compulsory and the second one is arbitrary which is done as needed. In the addressed method, before performing the two stages, another problem called pre-check should be solved to ensure the existence of leader-follower relationship in the problem so that the given problem can be considered as a BLP. Indeed, after confirming the result of pre-check test, the first stage will begin.

At the first stage of FGP, fuzzy membership degrees (FMDs) are formed for the objectives and LL's variables. Goals are then set for each FMD and considered equal to one, the highest value for every FMD. Afterwards, an FGP formulation is developed. A major limitation in FGP is that the satisfaction degree (SD) of the UL must be greater than that of the LL. FGP is then run, and the obtained solutions are employed for the second stage. If UL or LL do not accept the solutions, an adaptation scheme is run for the UL. The UL can reduce (or increase) its SD by increasing (or decreasing) the SD of the LL. Fuzzy variables corresponding to SDs take their values using linguistic expressions, which indicate the SD of the LL's fuzzy goal relative to the UL.

To implement the second stage, the linguistic expression is determined using SDs of the first stage; afterwards, by changing the discussed linguistic phrases, a new model is formulated in the second stage and its solutions are found. If the obtained solutions from the second stage are acceptable by the game players, the algorithm ends up; else, the second stage should be repeated in order to obtain an acceptable solution to both players. The drawback of doing the second stage is that it is time-consuming process.

According to the FGP-MCGP method of Partovi *et al.* (2023), new revisions in the FGP method are as follow: in the implementation of the FGP method, a single goal is not considered for each game player. In fact, new goals in the FGP are defined as continuous ranges. The benefit of this approach is that when UL expands his/her goal to a wider range, it permits LL to search in a wider range for acceptable solutions. Therefore, both players (UL and LL) will reach acceptable solutions in only one stage with faster solution times. In fact, by performing FGP-MCGP, the technique seeks top solutions within the given ranges of UL's SDs and LL's SDs, thereby unsatisfactory solutions are not found and BLP does not need to run the second stage.

4.1 Description of the FGP-MCGP Solution Method for the Given Problem

In this section, we describe the required steps for implementing FGP-MCGP method offered by Partovi *et al.* (2023) for solving the proposed problem. Equations (44)-(46) give the general formulation of the problem. Matrix A is an $m \times n$ matrix for showing the coefficients and C_i is a vector for representing the coefficients related to objective function i. The right-hand side values of the constraints are indicated by notation b. The set of decision variables is denoted by $x = \{x_1, x_2\}$, where x_1 represents the UL's decision variables and x_2 represents the LL's decision variables and Q shows the whole constraints set in the mathematical models.

$$\text{(UpperLevel): } DM_1\text{: } Max\ f_1(x) = C_1 x \quad (44)$$

$$\text{(Lower Level): } DM_2\text{: } Max\ f_2(x) = C_2 x \quad (45)$$

subject to:

$$x \in Q = \{x \in R^n | Ax \leq b, x \geq 0\} \quad (46)$$

In Equations (47)-(48), f_i^B (i =1, 2) represents the maximum (i.e., the optimum) value of the objective function i and its corresponding minimum (maximum) value is shown by f_i^W (i =1, 2).

$$f_i^B = f_i(\overline{x}_i) = \max_{x \in Q} f_i(x), \forall i = 1,2 \tag{47}$$

$$f_1^W = f_1(\overline{x}_2), f_2^W = f_2(\overline{x}_1) \tag{48}$$

Moreover, in Equations (47) and (48), the independent optimum solution of the leader (DM_1) is denoted by $\overline{x}_1 = (x_1^B, x_2)$ while for the follower (DM_2) is denoted by $\overline{x}_2 = (x_1, x_2^B)$.

According to the BLP formulations, we can consider constraints in (1)-(13), or we can consider them altogether as in the BLP model presented in Equations (44)-(46). Since any constraints for UL should be satisfied for LL, and vice versa, the obtained solutions by the developed method should be feasible for both LL and UL. We represent the formulation of SD for each level by Equation (49). The SD value is considered less than one; in fact, the SD represents the fuzzy membership degree of each objective function.

$$\mu_{f_i} = \begin{cases} 0 & \text{if } f_i(x, C_i) \le f_i^W, i = 1,2 \\ \frac{f_i(x,C_i) - f_i^W}{f_i^B - f_i^W} & \text{if } f_i^W \le f_i(x, C_i) \le f_i^B \;\; i = 1,2 \\ 1 & \text{if } f_i(x, C_i) \ge f_i^B, i = 1,2 \end{cases} \tag{49}$$

In Equation (49), $f_i(x, C_i)$ (i =1,2) represents the objective of DM_1 and DM_2 with decision variables $x = \{x_1, x_2\}$ and coefficients vectors C_i s. To attain more satisfying SDs for the LL, the UL can change his optimum decision variables for extending the search space for the LL. Two tolerance parameters indicated by q^R and q^L are considered for the optimum variables related to the UL, i.e., x_1^B; thus, the FMD of the UL's decision variable can be shown as in Equation (50).

$$\mu_{x_1} = \begin{cases} \frac{(x_1^B + q^R) - x_1}{q^R} & \text{if } x_1^B \le x_1 \le x_1^B + q^R \\ 1 & \text{if } x_1 = x_1^B \\ \frac{x_1 - (x_1^B - q^L)}{q^L} & \text{if } x_1^B - q^L \le x_1 \le x_1^B \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

Moreover, the pre-check test model is applied for checking the feasibility of $\mu_{f_1} \ge \mu_{f_2}$ constraint, as given in Equations (51)-(54).

$$\text{Min } \delta\delta \tag{51}$$

subject to:

$$\delta\delta \ge \mu_{f_2} - \mu_{f_1} \tag{52}$$

$$\delta\delta \ge 0 \tag{53}$$

$$x \in \mu_{x_1}^{-1}, \mu_f^{-1}, Q \tag{54}$$

In Equations (51)-(54), μ_f^{-1} and $\mu_{x_1}^{-1}$ denote the inverse functions of the vectors of the membership functions in Equations (49) and (50). If the value of $\delta\delta$ in the pre-check test model becomes zero, it can be concluded that the considered model has BLP structure, which means that the leader' SD is higher than the follower' SD.

According to the MCGP technique presented by Chang (2008), goals are defined by continuous intervals in Equations (58) and (61). This allows the LL to have a larger feasible space to search for higher SD solutions in one phase. The mathematical model of FGP-MCGP presented by Partovi et al. (2023) is given in (55)–(65).

$$Min(\sum_{i=1}^{2}[w_i(d_{f_i}^- + d_{f_i}^+) + \alpha_i(e_{f_i}^+ + e_{f_i}^-)]) + (w_1^{'}(d_{x_1}^- + d_{x_1}^+) + \alpha_1^{'}(e_{x_1}^- + e_{x_1}^+)) \tag{55}$$

subject to:

$$\mu_{f_i} - d_{f_i}^+ + d_{f_i}^- = y_{f_i}, i = 1,2 \tag{56}$$

$$y_{f_i} - e_{f_i}^+ + e_{f_i}^- = G_{f_i,max}, i = 1,2 \tag{57}$$

$$G_{f_i,min} \le y_{f_i} \le G_{f_i,max}, i = 1,2 \tag{58}$$

$$\mu_{x_1} - d_{x_1}^+ + d_{x_1}^- = y_{x_1} \tag{59}$$

$$y_{x_1} - e_{x_1}^+ + e_{x_1}^- = G_{x_1,max} \tag{60}$$

$$G_{x_1,min} \le y_{x_1} \le G_{x_1,max} \tag{61}$$

$$\mu_{f_1} \ge \mu_{f_2} \tag{62}$$

$$d_{f_i}^-, d_{f_i}^+, e_{f_i}^-, e_{f_i}^+ \ge 0, i = 1,2 \tag{63}$$

$$d_{x_1}^-, d_{x_1}^+, e_{x_1}^-, e_{x_1}^+ \ge 0 \tag{64}$$

$$x \in Q \tag{65}$$

In the FGP-MCGP model, $G_{f_i,max}$ and $G_{f_i,min}$ (i=1,2) in Equations (57) and (58) indicate upper bound (UB) and lower bound (LB) of the goals of the objective functions for the UL and LL. In the same way, $G_{x_1,max}$ and $G_{x_1,min}$ in Equations (60) and (61) indicate the UB and LB of the goals of the UL's decision variables. Note, it's the leader who can change a bit in the amounts of his/her decision variables in a continuous range to let LL for finding acceptable solutions in larger space. The notations y_{f_i} (i=1,2) and y_{x_1} are the continuous variables that change between $[G_{f_i,min}, G_{f_i,max}]$ and $[G_{x_1,min}, G_{x_1,max}]$ respectively. According to Chang (2008), $d_{f_i}^-$ and $d_{f_i}^+$ are the negative and positive deviations associated with ith goal $|\mu_{f_i} - y_{f_i}|$ with weight w_i, (i=1, 2), in Equation (55). In the same way, $d_{x_1}^+$ and $d_{x_1}^-$ are the positive and negative deviations associated with the goal of the UL's decision variables $|\mu_{x_1} - y_{x_1}|$ with weight w_1', in Equation (55). Indeed, the addressed deviation variables represent the deviation values from the best ideal values. In Equation (55), $\alpha_i (i = 1,2)$ and α_1' are weights of differences $|y_{f_i} - G_{f_i,max}|$ and $|y_{x_1} - G_{x_1,max}|$, respectively.

The obtained solutions as $x' = (x_1', x_2')$ from FGP-MCGP method in Equations (55)-(65), yield the highest values of SDs for both levels considering the higher priority for the UL than the LL.

4.2 The Algorithmic Steps of the FGP-MCGP Method for BLLIRP

The FGP-MCGP method described in section 4.1, is implemented in one stage to solve the flexible robust stochastic problem with satisfactory solutions for both LL and UL. The idea is that the FGP-MCGP method makes wider options for the DMs to find their optimal solutions. The steps of the FGP-MCGP method for our problem is briefly given here:

1) Step 1: Find the minimum and maximum values for each objective function $f_i(x)$ (i =1,2) for the two DMs, i.e., f_i^W and f_i^B using Equations (47) and (48).
2) Step 2: Obtain the FMDs of μ_{f_i} by using Equation (49) for the two DMs.

Table 3 The Values of Objective Functions and SDs for the Proposed Flexible Robust Stochastic BLLIRP Model

Example Number	Number of Warehouses	Number of Retailers	UL's Objective Functions Value UL's SD	LL' s Objective Functions Value LL's SD
1	2	2	TC_1=618.01 UL's SD_1 = %100	TC_2 =36.68 LL's SD_1 = %100
2	2	3	TC_1=611.16 UL's SD_2 = %100	TC_2 =58.03 LL's SD_2 = %100
3	3	6	TC_1 =619.05 UL's SD_3 = %99.98	TC_2 =531.65 LL's SD_3 = %97.81
4	3	8	TC_1 =612.61 UL's SD_4 = %99.98	TC_2 =731.5 LL's SD_4 = %93.83
5	4	11	TC_1 =341.63 UL's SD_5 = %100	TC_2 =562.62 LL's SD_5 = %100
6	4	20	TC_1 =341.63 UL's SD_6 = %100	TC_2 =1001.25 LL's SD_6 = %100
7	5	30	TC_1=334.81 UL's SD_7 = %100	TC_2=1482.35 LL's SD_7 = %100
8	5	40	TC_1=334.81 UL's SD_8 = %100	TC_2=1963.45 LL's SD_8 = %98.56

3) Step 3: find the best values of UL's decision variables x_1^B through independently optimizing the UL's objective function and then specify tolerances of q^R and q^L. Obtain the FMD of the UL's decision variables denoted by μ_{x_1} using Equation (50).
4) Step 4: To ensure the considered model is really a BLP, formulate and solve the pre-check test given in Equations (51)-(54). If the pre-check test confirms that the problem is BLP, then, go to Step 5; otherwise, stop.
5) Step 5: Formulate and solve the problem given in Equations (55)-(65) for finding the highest SDs for the both LL and UL.

5. COMPUTATIONAL RESULTS

In this section, we first design and solve some numerical samples for the proposed flexible robust stochastic BLLIRP model and apply the addressed solution method for solving it. Then, a sensitivity analysis on the two key parameters of the model is performed.

5.1 Numerical Examples

We have designed eight numerical examples to evaluate the performance of the proposed model and the efficiency of the FGP-MCGP solution method assuming different values for the parameters. The number of retailers (i.e., J) is considered 2, 3, 6, 8, 11, 20, 30 and 40 and the number of warehouses (i.e., I) is considered 2, 2, 3, 3, 4, 4, 5 and 5 for the eight different problems. Three scenarios, including pessimistic (called scenario 1), possible (called scenario 2) and optimistic (called scenario 3) scenarios with probabilities of occurrence equal to 0.3, 0.4 and 0.3 are considered. The demand of every retailer per unit time in scenario 1 (i.e., $\tilde{d}_{jp}^1$) is generated from interval [10, 140], in scenario 2 (i.e., $\tilde{d}_{jp}^2$) is generated from interval [15, 145] and in scenario 3 (i.e., $\tilde{d}_{jp}^3$) is generated from interval [20, 150]. The inventory holding cost of each retailer per time unit (i.e.,$\tilde{h}_{jp}^s$) is assumed to be generated from interval [2.5, 8]. The vehicle capacity (i.e., CV) is assumed to be 300, 250, 300, 600, 900, 1000, 1000 and 1000, for each example.

The number of vehicles (i.e., V) is assumed to be 3, 3, 4, 4, 5, 5, 6 and 7. The initial inventory (i.e., Ij0) is considered to be zero to all retailers. We have considered three time periods and the maximum storage life (i.e., πmax) is presumed to be 2 unit-times which means that the goods will spoil after two time periods after receiving in the retailers. The number of feasible routs (i.e., FR) are assumed to be 3, 2, 4, 5, 6, 6, 7 and 7 for the examples.

The cost of transportation (i.e., $\tilde{c}_r^s$) is considered proportional to the moving distance between each two-point and is assumed to be generated from interval [1, 7]. The tolerance rates q^R and q^L in the LL's decision variables are assumed to be one unit, because leader variables are binary. All similar weight coefficients in the objective function of the FGP-MCGP solution method in Equation (55) are considered to be equal. We have presumed all the trapezoidal fuzzy numbers related to the degrees of violation of flexible constraints to be identical and equal to (1,2,3,4) for $\tilde{k}$= (k1, k2, k3, k4) and $\tilde{t}$= (t1, t2, t3, t4). It should be noted that the deviation values of the flexible constraints are proportional to the amount of data. Since most of the used data for demand and order quantities are less than 50, so the amount of violation is considered around 5 %.

LB and UB goals for the leader and follower are considered as $G_{f_1,\min} = G_{f_2,\min} = 0.8$ and $G_{f_1,\max} = G_{f_2,\max} = 1$. The same values for the leader's decision variables are assumed to be in the interval of [0.8, 1]. This interval seems reasonable and acceptable for the SDs of both leader and follower.

The values of pessimistic-optimistic parameters of the leader and follower, (i.e., λ_1 and λ_2), in the general fuzzy measure approach (Me measure) are assumed to be identical (i.e., λ_1=λ_2) and equal to 0.75, 0.75, 0.85, 0.85, 0.9, 0.9, 0.9 and 0.9, respectively.

We have utilized Lingo 17 optimization package for solving the problem. The pre-check test presented in Equations (51)-(54) has been done for all samples and the values of the objective function for all the samples are obtained zero (that is, δδ = 0); thus, all numerical examples are of BLP type. The value of the objective function for each level (i.e., UL and LL) and their corresponding SDs using the proposed FGP-MCGP solution method are given in Table 3. Since the SD percentages for the LL are equal to 100% and for the follower are more than 93% in all samples, it can be concluded that the results are satisfactory for both sides of the game. The values of the decision variables for the first example (with two warehouses and two retailers) are reported as in Table 4.

Table 4 The Values of the Decision Variables for the First Numerical Example with FGP-MCGP Solution Method

LL's Decision Variables		UL's Decision Variables	
I_{jp}^{s}	Q_{jrp}^{s}	θ_{rp}^{s}	m_i
All = 0	Q_{111}^1=30, Q_{111}^2=33.5, Q_{111}^3=38.5, Q_{112}^1=40, Q_{112}^2=45, Q_{112}^3=50, Q_{113}^1=140, Q_{113}^2=105, Q_{113}^3=110, Q_{211}^1=20, Q_{211}^2=25, Q_{211}^3=30, Q_{212}^1=50, Q_{212}^2=45, Q_{212}^3=50, Q_{213}^1=40, Q_{213}^2=45, Q_{213}^3=50 The rest = 0	θ_{11}^1, θ_{11}^2, θ_{11}^3, θ_{12}^1, θ_{12}^2, θ_{12}^3, θ_{13}^1, θ_{13}^2, θ_{13}^3=1 The rest= 0	m_1=1 m_2=0

Table 5 Individual Maximum and Corresponding Minimum for Two Objective Functions in the Eighth Numerical Sample

	$f_1(x)$	$f_2(x)$
f_i^W	-2834.8	-6669.57
f_i^B	-334.81	-1895.02

According to the results given in Tables 3 and 4, in the first numerical example, when the LL opens the first warehouse and employs route 1 for all time periods and for all scenarios, the SD reaches 100%. The follower reaches the same SD, i.e., 100%. In Table 4, the values of Q_{jrp}^{s} shows that the only selected route is route 1 in time period 1, 2 and 3 and scenarios 1, 2 and 3. With these values of decision variables for the UL, the LL does not need to keep inventory to meet the demand, and all retailers do not pay inventory holding costs.We provide more details of the solution steps for the eighth numerical example in Table 3, where there are five warehouses and forty retailers. The DM in each level, i.e., DM_i (i = 1, 2), has to optimize his objective function and only determines his decision variables. For the leader (DM_1), the individual maximum value of the objective function f_1^B and its corresponding minimum value f_1^W are calculated employing Equations (66) and (67) and Equations (28)-(43). Note that we have converted the objectives of both levels from minimization to maximization.

$$f_1^B = \max_{x\in Q} f_1(x) = f_1(\overline{x}_1) \text{ and } f_1^W = f_1(\overline{x}_2) \tag{66}$$

$$f_2^B = \max_{x\in Q} f_2(x) = f_2(\overline{x}_2) \text{ and } f_2^W = f_2(\overline{x}_1) \tag{67}$$

The obtained values are given as in Table 5.

The satisfaction degree of each DM (i.e., μ_{f_1} and μ_{f_2}) is obtained from Table 5 by applying Equation (49). For DM_1, the optimized solution with the objective function f_1^B is denoted by $\overline{x_1}$= (x_1^B, x_2). In the mentioned solution, x_1^B is as follows:

$m_4 = 1, m_2 = m_3 = m_1 = m_5 = 0$ and $\theta_{61}^1 = \theta_{61}^2 = \theta_{61}^3 = \theta_{62}^1 = \theta_{62}^2 = \theta_{62}^3 = \theta_{63}^1 = \theta_{63}^2 = \theta_{63}^3 = 1$ and the rest of θ_{rp}^s are zero.

Having obtained x_1^B and assuming that the left and right tolerances of x_1^B to be one unit (i.e., $q^R = q^L = 1$), the FMDs of the leader's variables are obtained by employing Equations (50). The pre-check test model in (51)-(54) is run and its objective function turns out to be zero (i.e., $\delta\delta$ =0); thus, the problem is of BLP. Then, the FGP-MCGP method in Equations (55)-(65) is implemented to find the optimum solution $x' = (x_1', x_2')$; as for DM_1, the decision variables, i.e., x_1', are as follows:
$m_4 = 1, m_2 = m_3 = m_1 = m_5 = 0$ and $\theta_{61}^1 = \theta_{61}^2 = \theta_{61}^3 = \theta_{62}^1 = \theta_{62}^2 = \theta_{62}^3 = \theta_{63}^1 = \theta_{63}^2 = \theta_{63}^3 = 1$ and the rest of θ_{rp}^s are zero.

The SDs for the two DMs are obtained as μ_{f_1}= %100 and μ_{f_2} = %98.56, which satisfy the condition of the BLP problem (i.e., $\mu_{f_1} \geq \mu_{f_2}$). The values obtained for over and under deviation variables in the FGP-MCGP method are $d_{f_1}^+$ =0.00000002, $d_{f_2}^+$ =0, $d_{f_1}^-$ =0, $d_{f_2}^-$ =0.0143, and all the values of $d_{x_1}^+$ and $d_{x_1}^-$ are equal to zero. The leader and the follower are satisfied with the values of the satisfaction degrees of 100 percent and 98.56 percent.

5.2 Sensitivity Analysis

In this section, the sensitivity analysis is performed considering pessimistic and optimistic attitudes of the DMs by selecting the value of parameter of λ in Equations (28)-(43). Since we have two DMs, instead of one parameter of λ, two parameters of λ_1 and λ_2 are considered for the pessimistic-optimistic attitudes of UL (DM_1) and LL (DM_2). The second numerical example in Table 3 with two warehouses and three retailers is considered for the analysis. The values of the objective functions and their SDs for both UL and LL are reported in Table 6. The value of λ_1 changes from zero to one, while the value of λ_2 remains fixed at 0.75. The results of the objective functions and SDs are reported in Table 6 for the both UL and LL. Figure 3 shows the variations in the objective functions when changing λ_1.

From Table 6 and Figure 3, it is clear that increasing the value of λ_1 results in growing the costs of the UL from 509.62 to 645.01. SD values remain constant at %100 for all cases; the justification is that it is dependent from λ_1.

As for the follower, the values of pessimistic-optimistic attitudes of the LL (i.e., λ_2) changes from 0.7 to 0.99, while the value of λ_1 is fixed at 0.75. It should be noted that for the exact value of λ_2=1, the mathematical problem becomes undefined due to the phrase in denominator (λ_2-1), and for values of λ_2 less than 0.7, the problem becomes infeasible.

The upper bound of λ_2 can be determined easily, because according to Equations (34)-(38), 1-λ_2 is in the denominator and it should be 1-λ_2>0 which results in 1 >λ_2. For this purpose, in the sensitivity analysis, we have started from 0.99. To determine the lower bound of λ_2, we need to consider Equation (35) in the constraints of the proposed model that corresponds to the inventory balance constraint.

In Equation (35), for the case of one retailer, three time periods and three scenarios (i.e., j = 1, p = 3 and s = 3), the demands and order quantities take their highest values. Putting the values of different parameters in Equation (35), the lower bound of λ_2, can be found as in (68).

Table 6 Results of the Sensitivity Analysis Considering Changes in λ_1 with λ_2=0.75

λ_1=	0	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1
Leader's Cost	509.62	529.93	543.47	557.01	570.54	584.08	597.62	611.16	624.7	638.24	45.01
Leader's SD	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100
Follower's Cost	58.03	58.03	58.03	58.03	58.03	58.03	58.03	58.03	58.03	58.03	58.03
Follower's SD	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100

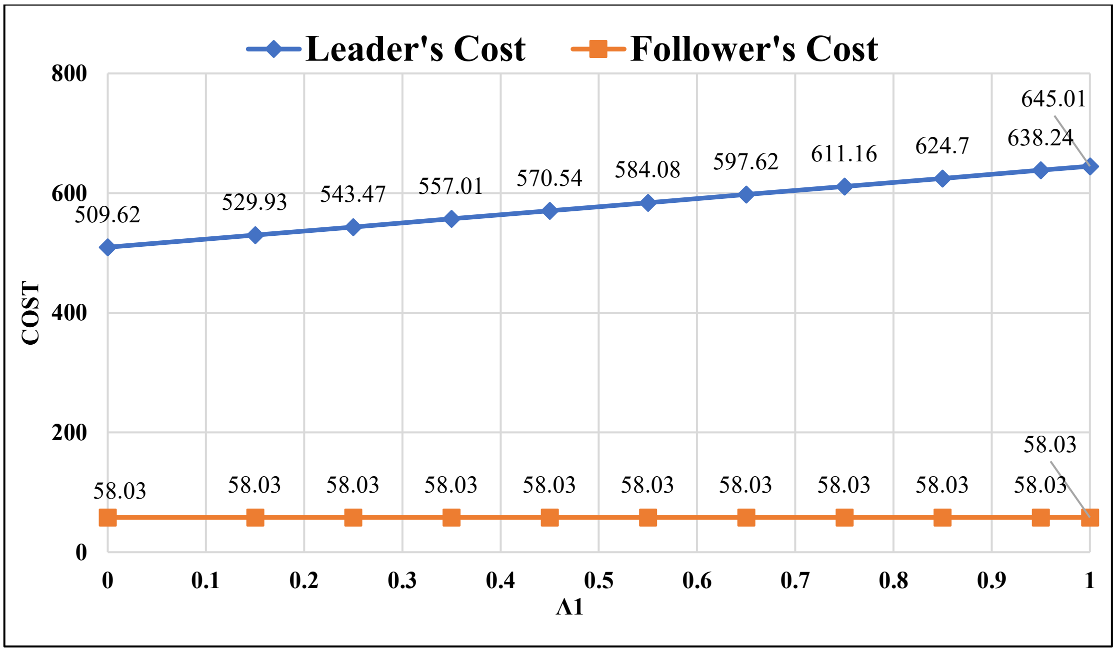

Figure 3 Changes in the Leader's and Follower's Cost Due to Changes in λ_1

$$I^s_{jp-1} + \sum_{r \in FR} aa_{jr}\, Q^s_{jrp} \geq \left(\frac{(\delta_s - \lambda_2) d^s_{jp4} + (1 - \delta_s) d^s_{jp3}}{1 - \lambda_2} \right)$$

$$+ I^s_{jp}, \forall p \in P, \forall j \in J, \forall s \in SC$$

$$116.6667 \geq \frac{(0.5 - \lambda_2)(150) + 0.5(130)}{1 - \lambda_2}$$

$$116.6667 - 116.6667\lambda_2 \geq 140 - 150\lambda_2$$

$$\lambda_2 \geq \frac{23.3333}{33.3333} \cong 0.7$$

$$\lambda_2 \geq 0.7 \tag{68}$$

we have already stated that $\lambda_2 < 1$, so the feasible of λ_2 is: $0.7 \leq \lambda_2 < 1$.

The results for the objective functions including costs and SDs are reported in Table 7 for the both UL and LL. Figure 4 shows the changes in the objective functions of the leader and follower when λ_2 changes.

From Table 7 and Figure 4, it is clear that by increasing λ_2, a slight decrease in the LL's costs from 58.96 to 53.59 occurs. SD values remain constant at %100 for all cases, because independent from $\lambda 2$. Therefore, increasing or decreasing λ_2 will not affect the SDs. For the UL, since the parameter λ_2 does not play any role in its equations, the cost function and SD remain constant.

By increasing λ_1, the leader's total cost grows due to the possibility of higher cost parameter values, while it has no effect on the follower's costs due to its independence from λ_1. The results in Table 6 confirm this. Conversely, increasing λ_2 partially decreases the follower's total cost, while the leader's total cost remains constant. This reduction in the follower's total cost is due to equations related to constraints associated with uncertain parameters with coefficients ω and ϑ. In fact, increasing λ_2 may raise the average value of the follower's total cost, but due to other terms in the feasibility robustness equations, the total cost does not increase. This is confirmed by the results in Table 7. Furthermore, the leader's objective function has no additional term for feasibility robustness similar to that of the follower; thus, increasing λ_1 increases the leader's total cost.

Another sensitivity analysis that is performed in this research is on parameters ψ_1 and η_1 in Equations (28). Initially, we should make clear that we have chosen the weight values of the robust terms of the leader and follower objective functions in Equation (28) and (34) in such a way that their sum to be equal to one for the leader and follower separately. On the other hand, the sum of the values of η_1 and ψ_1 in the leader objective function are equal to 1 (initially, each one is 0,5) and the values of η_2 and ψ_2 in the follower objective function are equal to 0.3 while the other weights including ω, ϑ, γ, and Θ are considered equal to 0.1 In Fact, we have considered equal importance for the optimality

Table 7 Results of the Sensitivity Analysis Considering Changes in λ_2 with λ_1=0.75

λ_2=	0.7	0.73	0.75	0.78	0.80	0.85	0.88	0.90	0.95	0.99
Leader's cost	611.16	611.16	611.16	611.16	611.16	611.16	611.16	611.16	611.16	611.16
Leader's SD	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100
Follower's cost	58.96	58.40	58.03	57.48	57.11	56.18	55.63	55.26	54.33	53.59
Follower's SD	%100	%100	%100	%100	%100	%100	%100	%100	%100	%100

Table 8 Results of the Sensitivity Analysis Considering Changes ψ_1 and η_1

ψ_1 and η_1=	ψ_1=1 η_1=0	ψ_1=0.9 η_1=0.1	ψ_1=0.8 η_1=0.2	ψ_1=0.7 η_1=0.3	ψ_1=0.6 η_1=0.4	ψ_1=0.5 η_1=0.5	ψ_1=0.4 η_1=0.6	ψ_1=0.3 η_1=0.7	ψ_1=0.2 η_1=0.8	ψ_1=0.1 η_1=0.9	ψ_1=0 η_1=1
Leader's Cost	716.65	695.55	674.46	653.36	632.26	611.17	590.07	568.97	547.88	526.78	505.69
Leader's SD	%87	%90	%92	%95	%97	%100	%100	%100	%100	%100	%100
Follower's Cost	59.94	59.56	59.18	58.80	58.41	58.03	58.03	58.03	58.03	58.03	58.03
Follower's SD	%87	%90	%92	%95	%97	%100	%100	%100	%100	%100	%100

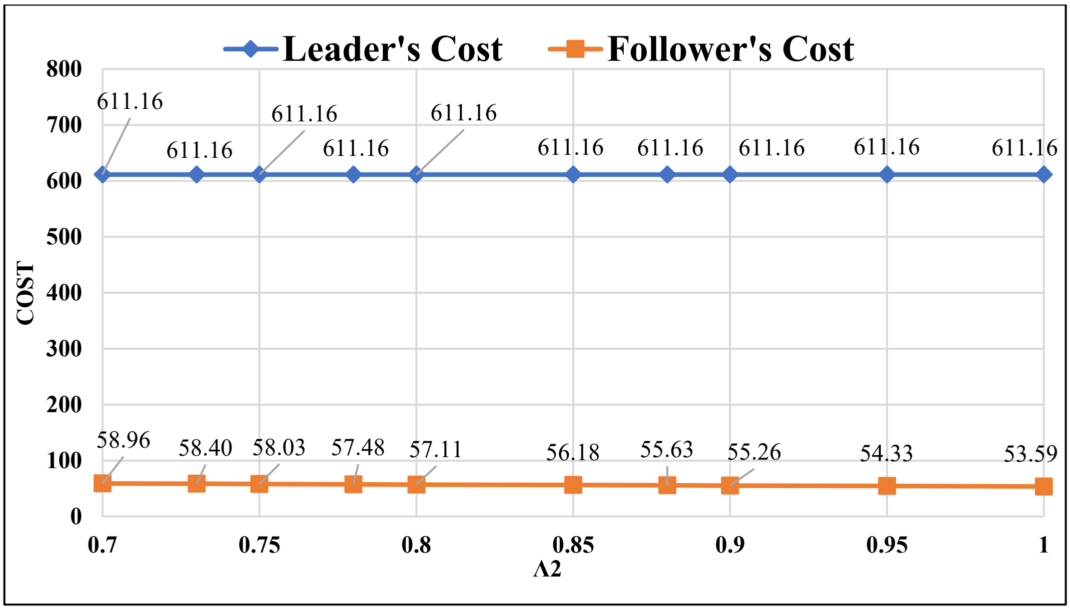

Figure 4 Changes in the LL's and UL's Costs when λ_2 Changes

robustness under fuzzy parameters and optimality robustness under scenario-based parameters. The second numerical example in Table 3, with two warehouses and three retailers, is considered for analysis. The values of the objective functions and their SDs for the UL and LL are reported in Table 8. Figure 5 shows changes in the objective functions and SDs of both levels when changing ψ_1 and η_1.

According to Table 8 and Figure 5, by decreasing ψ_1, which corresponds to the optimality robustness under fuzzy parameters in the leader 's objective function and by increasing η_1 (considering that $\psi_1 + \eta_1 = 1$) , which corresponds to the optimality robustness under scenario-based parameters, the leader 's costs decreases and its SD increases by the maximum value of one. On the other side, the addressed changes in ψ_1 and η_1, reduces the follower' cost and increases its SD. It is clear that initial values of the parameters ψ_1 and η_1, which lead to the highest value of satisfaction degrees (i.e., SD=1) for the leader and follower, are 0.5 and 0.5. For this reason, we have let their initial value equal to 0.5.

6. CONCLUSIONS AND FUTURE IDEAS FOR RESEARCH

In this research, we proposed an uncertain mathematical model for a BLLIRP problem called robust flexible stochastic BLLIRP. The under-study problem is in a two-echelon SC consisting of a few warehouses as the leader and several retailers as the follower who receive a perishable product from the leader. Some constraints of the model were assumed to be flexible which leads to limited violation. We have employed the general fuzzy Me measure to embed the pessimistic-optimistic attitudes of the DMs. Furthermore, scenario-based robust programming was utilized for constraints with uncertain parameters. The decisions for the warehouses were determining the best locations and selecting the appropriate routes to distribute goods to retailers in different time periods and scenarios while for the retailers, decision variables were the number of inventories received in different time periods and inventory held.

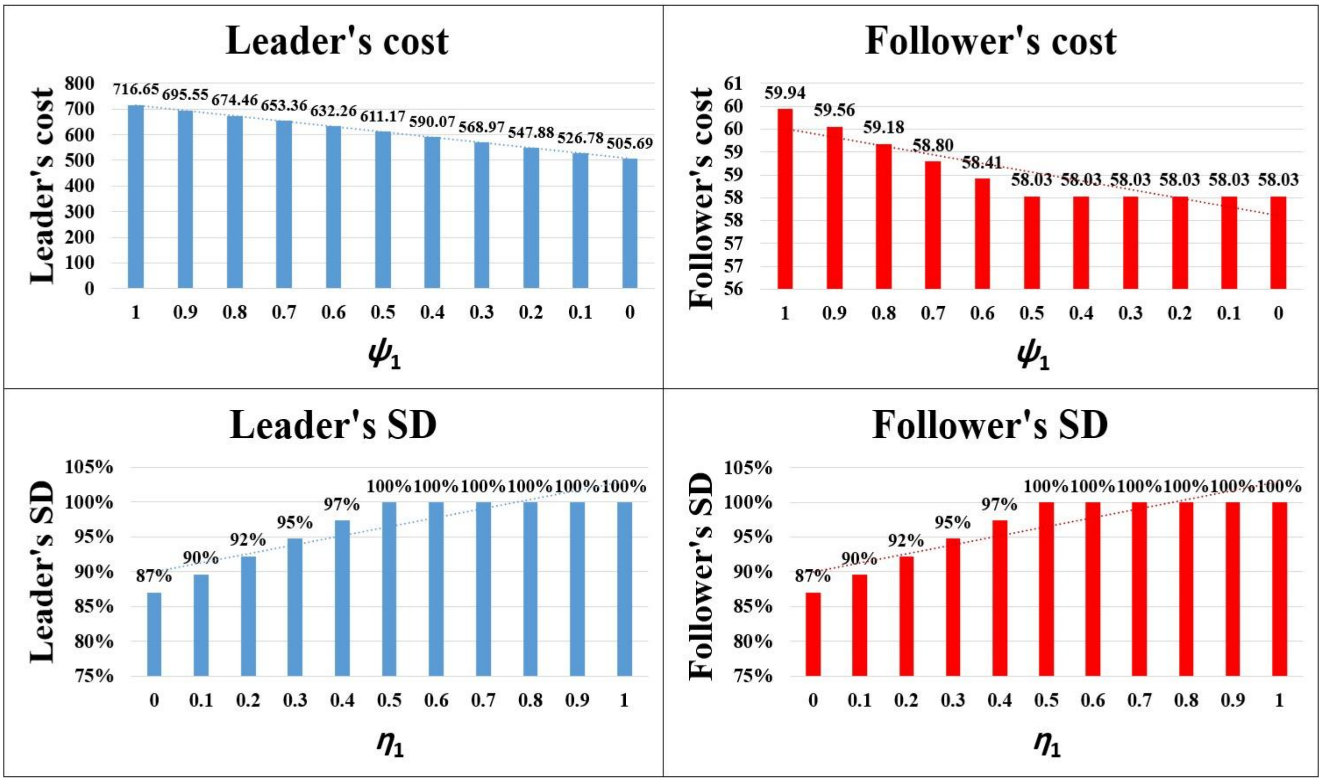

Figure 5 Changes in the LL's and UL's Costs and SDs when ψ_1 and η_1 Change

As for the solution, the FGP-MCGP solution method of Partovi *et al.* (2023) was customized to solve the addressed problem. To evaluate the FGP-MCGP solution method efficiency, a few numerical examples with different sizes were solved and the results of the objective functions of both levels and the corresponding SDs were reported. Sensitivity analysis on the pessimistic-optimistic attitudes of the UL and LL was performed, as well. In generating the numerical problems, we took great care in fitting the values of the various parameters so that the generated problems were as close to real-world values as possible. However, not directly utilizing the data of a real case study can be known as the research limitation of this study and future researches can implement the model using the real-world data in perishable items distribution networks such as pharmaceutical and dairy products.

As for managerial insights, the proposed model can be applied to many distribution-based two-echelon SCs in which in the first echelon, there are distribution centers/ warehouses and in the second echelon, there are retailers which receive a perishable item from the warehouses in food or pharmaceutical industries. It is clear from Equations (28) and (34) that increasing the value of λ1 and λ2 means that the UL and LL move to the side of being pessimistic rather than being optimistic. The total cost of the UL increases as surging the value of λ1 for a given value of λ2. The key result is that if the leader ensures the stability of the follower's attitude towards risk (noting that this is possible for the leader due to the role of leadership), it can be sure that its operational costs will increase in case of increasing the leader's pessimistic-optimistic attitude, i.e., λ1. For the given example, it is clear from Table 6 that the total cost of the leader increases from 611.16 to 638.24 when λ1 increases from 0.75 to 0.95. Thus, the key managerial implication for the leader is that it can control its operational costs (including fixed costs of opening warehouses and transportation costs in the selected routes) when making sure of the stability of the follower's attitude toward risk.

Furthermore, as the optimality robustness under fuzzy parameters decreases in the leader 's objective function and correspondingly, the optimality robustness under scenario-based parameters increases, the leader 's costs decreases and its SD increases by the maximum value of one.

As future research ideas, the developed model can be extended to multi-product fast-moving consuming goods SCs with uncertain parameters. Furthermore, the multi-product problem with different shelf-lives can be investigated. Another idea is to consider inventory holding costs at both warehouses and retailers. Demand can be considered dependent on advertising campaigns, product price and greening level of product. Environmental effects and social factors can be considered in decision-making such as using different vehicles with various pollution amounts and job creation levels. Retailers can be in a competitive market, and the problem can be formulated in multi-follower environment.

CONFLICTS OF INTEREST

The authors have no relevant financial or non-financial interests to disclose.

DATA AVAILABILITY STATEMENTS

All data that used in this study are provided in the paper and there is no other attached data to this manuscript. Furthermore, the computer code of LINGO can be accessed via the following permanent link:
https://drive.google.com/file/d/1kDTelKf0rSZkq6nRWGeNiO7eiJRpktf3/view?usp=drive_link

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